Doppler structure in echoes from schools of pelagic fish

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The frequency distribution of energy in direct path echoes from three schools of pelagic fish is presented. One-half-second 30-kHz CW pulses were used to study motions internal to the fish schools from distances up to 1200 m. The measured Doppler spread of echo energy ranged from 30 Hz to 70 Hz. The Doppler spread-at-side aspect is related to swimming motions by a simple algebraic model based on Bainbridge’s equation relating fish swimming speed, length, tail-beat amplitude, and tail-beat frequency. The mathematical model was used to estimate the length of the fish in two of the schools. These length estimates agree with average fish lengths derived from school cruising speeds. Near head or tail aspect, the observed Doppler structure appears to be related to behavioral swimming characteristics.

INTRODUCTION

Based upon acoustic measurements, and visual observations, it is apparent that a school of fish is a dynamic system characterized by varying degrees of systematic as well as random motion. The scale of movements in a school ranges from rhythmical, swimming-associated body motions of individual fish, including tail movements, to erratic, rapid changes in speed, direction, and shape of the entire school. Between these extremes lie a variety of motions, including cruising, swimming bursts, short-term accelerations, the constant changing of relative positions of individuals in a school, and several behavioral characteristics.

Information on fish length, swimming endurance, and school dynamics are all contained in the motions associated with a school. Moreover, an appropriate tool exists for the study of school motions. It takes the form of a measurement of Doppler frequency shifts in acoustic echoes from a fish school. Although the measurement of Doppler structure has been frequently discussed, it has been infrequently attempted and with even less frequent success.

The following material contains a description of the experimental method for measuring the Doppler structure in echoes from fish schools. The frequency spectra of narrow-band echoes from three pelagic fish schools are presented. Additional acoustically derived data on the three schools, an analysis of the data, and, where possible, relation of the acoustic data to biologically meaningful parameters are also presented. The three schools were studied in late July 1972 at the locations shown in Fig. 1.

1. EXPERIMENT

In the current application, the Doppler effect refers to the change in frequency (pitch) which occurs when sound waves are reflected from a moving object. A frequency shift is similarly observed when sound is projected from or received by a moving platform. The shift is toward higher frequencies if the relative motion of the receiver and reflector is closing, and down in frequency if opening. The amount of the frequency shift is proportional to the original frequency and to the relative speed of approach or regression of the receiver (projector) and reflector.

Conceptually, one needs to transmit a pulse sufficiently narrow in bandwidth to resolve the degree of motion expected and then determine the distribution of energy in the echo as a function of frequency. The distribution of echo energy can then be related to a distribution of motions (velocities along the direction of propagation of sound) in the school of fish.

Though simple in concept, the execution of an experiment designed to make quantitative measurements of the Doppler structure in echoes from fish schools is sufficiently complex to make a computer-controlled data acquisition system desirable, if not a necessity.

The data acquisition system illustrated in Fig. 2 was designed around a 12-bit minicomputer with a memory cycle time of 1.6 μsec and 8192 words of core storage. The system was interfaced to an existing hull-mounted 30-kHz sonar on the NOAA Research Vessel DAVID STARR JORDAN. The JORDAN’s 11-kHz sonar was used to collect ancillary data on the track of the fish school. The vertical and azimuthal beam widths of each sonar were approximately 12° at the 3-dB points. Both sonars could be trained through 360° in azimuth and the 30-kHz transducer was vertically steerable. The 11-kHz source level was 220 dB while the 30-kHz source level was 224 dB, both relative to 1 μPa at 1 m. A high-level interpretive software system, originally supplied by the computer manufacturer, was substantially modified and expanded to be used in controlling the experiment. This approach allowed both rapid, algebraic-like interaction with the data acquisition process (e.g., updating range, speed of sound, etc.), as well as in situ modifications of the experimental method, which would not have been practical if the software had been coded directly in machine language.

In a typical sequence, a fish school was detected with the 11-kHz sonar at a range of 300 to 700 m from the ship. The ship was stopped and allowed to drift with wind and current. There were two reasons for stopping the ship. The first was a desire to study single fish schools for extended periods, for example, an hour or more. Contact with a fish school from a ship moving at
6 to 12 knots along a straight track lasts no more than a few minutes. The second reason for allowing the ship to drift was related to equipment limitations. The 30-kHz sonar was originally designed to transmit pulses of up to 80-msec duration. It was necessary to slow the pulse repetition rate to one pulse each 100 sec in order to use the ½- and 1-sec pulses required for measuring Doppler structure. At faster pulse repetition rates, noticeable degradation of the 30-kHz sonar source level was apparent. The 100-sec interval dictated that the ship–fish geometry change as slowly as possible in order that necessary ensemble averaging could be performed without catastrophic violations of the requirement of stationarity. Drifting, as opposed to low-speed maneuvering, was determined to be the practical solution to this problem.

The drift rate and direction, with respect to the water column at the fish range and depth, were accurately determined on a periodic basis by measuring the direction and amount of the maximum Doppler shift of volume reverberation. The primary contributor to volume reverberation is plankton which drifts with the body of water. Thus, an accurate measure of the ship's motion relative to the water column is readily available. This is essential if accurate measurements of fish school track and swimming speed are to be made. The presence of a stable reference, such as the frequency of the volume reverberation, also allows a first-order compensation for the effects of making Doppler structure measurements from a moving platform rather than a stationary one.

Tracking of the target was started with the 11-kHz sonar, aided by a programmable calculator, as soon as the ship stopped. When a steady drift rate and direction were established, the computer was instructed to generate and transmit, via the 30-kHz sonar, a pulse of ½-, ½-, or 1-sec duration, depending on the target range. The data presented later in this paper are from the ½-sec transmissions.

At the end of the 30-kHz transmission, the signal generator frequency was switched to 29.9 kHz. This signal was then used to heterodyne the received signal, spread about 30 kHz by the Doppler effect, to a band between 0 and 200 Hz. The sum band was rejected by a low-pass filter. At a time determined by the computer from the target range and average speed of sound over the ray path to the school, a command was issued to generate a range gate in the form of one-half of one cycle of a cosine function. This form of range gate was combined with similar temporal weighting in the spectrum analyzer to suppress side lobes in the frequency domain.

Again, under computer control, a command was issued to capture the fish school echo in a spectrum analyzer memory. The memory was automatically displayed on a CRT. Immediately after the echo was captured, its

FIG. 1. Location of schools studied in July 1972.
spectrum was digitized and transferred from the spectrum analyzer to the computer for display on a second CRT, and for output to punched paper tape for later analysis. An option to accumulate and display an ensemble average was available but was infrequently used due to the length of time for on-line computer corrections for the effects of the ship's motions.

All displays were made available for examination until the 100-sec-waiting interval had elapsed. At the end of that time, the spectrum analyzer was released to accept new data and the sequence repeated. The sequence was occasionally interrupted for a period of less than 10 sec for an update on school range.

During the 100-sec delay between 30-kHz pings, the 11-kHz sonar was used to obtain bearing and range information. A minimum of 10 pings at 2-sec intervals was used to determine the direction and range of the school on the basis of maximum acoustic return. The 11-kHz sonar was not operated simultaneously with the 30-kHz sonar because of a problem of mutual interference.

II. DATA

A. Data format

The echo from a fish school is stochastic. Consequently, the Fourier transform of a single echo is likewise stochastic. In order to obtain a physically more meaningful representation of the frequency distribution of energy scattered from a school, the energy spectral density was computed by performing an ensemble average over several sequential echoes. The indicated calculation was performed as a running average over seven successive pings throughout the available data sequence for each of three schools.

Aspect and school swimming speed relative to the water column are included in the caption for each figure displaying a sequence of energy spectra as a function of Doppler frequency shift. A more detailed description of aspect and speed are contained in the discussion of each set of spectra. Aspect and swimming speed numbers associated with a particular energy spectrum are averages taken over the same time interval as the ensemble average used to compute the spectrum. An unshifted
Target A was detected at approximately 9 a.m. on 21 July 1972 in about 325 fathoms water depth, approximately 4 miles southwest of Laguna Beach, California. The target dimension along the direction of the sonar beam varied from 60 to 120 m during the period of observation. Measurements of target sizes were made with 1-msec, 11-kHz pulses.

Figure 3 illustrates a sequence of nine energy spectral density curves for echoes from Target A. Each curve represents a running average over seven echoes. During the time of acquisition of the data shown in Fig. 3, the sonar beam was, on the average, nearly orthogonal to the swimming direction of the fish school. The average swimming speed during this interval was 0.85 m/sec.

An intense narrow return near zero Doppler shift is apparent in all spectra presented. This return is due to volume reverberation. Each spectrum has been shifted in frequency before averaging, to align the volume reverberation in frequency and to place the reverberation frequency at the mean frequency of the volume return in the set averaged. This process was used as a first-order correction for the Doppler effects due to the ship's oscillatory motions, principally roll. Since the sonar beam and the direction of school travel were approximately orthogonal, the echo from the fish school appears symmetrically distributed about the volume reverberation return. The frequency of the volume reverberation is a convenient reference of no motion, relative to the water column. The echo in the spectrum numbered 1 is present but very weak. Therefore, the spread of energy at the base of the volume reverberation return in spectrum 1 can be used to indicate the small contribution of side lobes (due to the finite pulse length) to the spectral level in the vicinity of the volume reverberation return.

A close examination of spectra numbers 5–9 in Fig. 3 will reveal a return with a Doppler shift of about –100 Hz. This return is due to bottom reverberation. As a sequence of echoes was obtained from a school, both water depth and the range to the target varied. Therefore, for some combinations of water depth and range gate settings, reverberation from the bottom appears at a frequency determined by the ship's drift (wind) with respect to the water and the movement of the water column with respect to the bottom (currents).

A sequence of near-tail aspect echoes from Target A is presented in Fig. 4. The aspect varied in a linear manner from 30° (near-tail aspect) at spectrum 21, to about 5° from tail aspect at spectrum 29. The school
swimming speed was almost constant at an average 1.05 m/sec during the interval between spectra 21 and 29.

The echo is seen in Fig. 4 as a distribution of energy centered about a negative Doppler shift which varies from about 35 Hz at spectrum 21, to 40 Hz at spectrum 29. The changing Doppler shift of the echo is principally due to the change in the sonar beam axis with respect to the school travel direction.

C. Target B

This 40-m-wide target was located at approximately 11 a.m. on the north flank of the Lasuen Seamount. The target was at a depth of approximately 40 m. The water depth was 250 fathoms.

A sequence of energy spectra from Target B is shown in Fig. 5. During the interval for which the spectra are
provided, the school aspect varied less than $5^\circ$ from a tail-quarter aspect. At the time of spectrum 11, the aspect was $40^\circ$. The aspect increased linearly to $50^\circ$ at the time of the center echo in the ensemble average which resulted in spectrum 16, and remained at $50^\circ$ until the end of the data sequence. The school swimming speed remained near 1.4 m/sec between spectra 11 and 16. A decrease in swimming speed to 0.5 m/sec occurred between spectra 16 and 19. The speed remained constant at 0.5 m/sec from spectrum 19 to the end of the data sequence.

The echo from the fish school can be seen in Fig. 5 as a distribution of energy centered about a lower frequency than the water column reference, the volume reverberation return. The volume reverberation appears at a positive Doppler shift due to the ship's drifting speed and the relative angles between the direction of drift and the sonar beam axis. The frequency separation between the volume reverberation and the center of the fish echo energy distribution can be used to measure the component of fish school speed along the sonar beam axis. Calculations of school swimming speed from the component of motion along the sonar beam and the track-derived angle between the sonar beam and the swimming direction of the school exhibit the expected agreement in speed based on school track data alone.

The increase in echo level between spectra 11 and 15 was associated with an apparent compaction of the school and with the observed change in swimming direction. The increase in echo level is also consistent with the change in aspect angle from near $40^\circ$ to $50^\circ$, i.e., $10^\circ$ nearer side aspect.

D. Target C

Target C was located at approximately 1 p.m. on the same day in the San Pedro Basin near Santa Catalina Island. The water depth was about 475 fathoms. The target size remained approximately 60 m throughout the period of observation. This fish school was also estimated to be near a depth of 40 m. This estimate, as well as those for the two previous schools, was derived by comparison of the target acquisition range with the acoustic propagation path. The propagation path was computed at sea from temperature profiles of the water column taken near the school locations during the same period.

Both aspect and swimming speed remained nearly constant during the interval covered by the data in Fig. 6. The fish were swimming almost directly at the ship during the entire period of observation. The ship, however, was drifting away from the fish at a greater rate than the fish swimming speed. Consequently, although both volume reverberation and the echo from the school are shifted down in frequency, the distribution of echo energy appears at a higher frequency than the water motion reference, volume reverberation. The difference of about +20 Hz between the volume reverberation return frequency and the echo frequency is consistent with the track derived swimming speed of 0.45 m/sec.

The return near-100-Hz Doppler shift is bottom reverberation. As the period of observation progressed, the ship drifted into slightly deeper water and the amount of bottom reverberation coincident with the echo was reduced.
III. ANALYSIS AND INTERPRETATION

A. Fish size and cruising speed

Each of the schools for which Doppler spectra are presented was observed for about one hour. A detailed analysis of the swimming speed reveals periods of relatively constant swimming speeds. An average length can be computed for the fish in each school if one assumes that the observed constant swimming speeds correspond to cruising or sustained swimming speeds described by Bainbridge and by Mangan. Measured sustained speeds for fish generally range between 2 and 4 body lengths/sec. The best available estimate of the low cruising speed of marine species appears to be the work of Mangan in 1930. The average cruising speed for 17 marine species common to the North Atlantic was 3.2 body lengths/sec.

Based on cruising speeds of 0.9, 0.85, and 0.45 m/sec for Targets A, B, and C, respectively, one can calculate the length of an average fish in each school. The lengths which result from this calculation are 0.29 m for Target A, 0.27 m for Target B, and 0.14 m for Target C.

B. Fish size and Doppler structure

A detailed examination of fish school tracks and swimming speed records indicate that rapid accelerations and decelerations within a fish school, and accompanying changes in school shape due to a lag in response between the fish in the front of the school and those in the back may be common. Under these conditions, one would expect a relatively wide distribution of swimming speeds at a particular instant during a transient in the school's speed of advance. In order to reduce the impact of these behavioral movements on the estimate of fish size from body and tail motions, we have restricted our size calculations to cases in which the fish presented a side aspect to the sonar beam. In this manner, the Doppler spread due to a distribution of swimming speeds along the direction of school advance was minimized, and the effect of body motions perpendicular to the direction of school headway was enhanced.

In a study of three species of fish, Bainbridge determined that the swimming speed (V) of all three species could be calculated from their total length (L) and their tail-beat frequency (F) by the relation of Eq. 1.

\[ V = \left( \frac{L}{4} \right) (3F - 4) \]  

The swimming speed may be expressed in meters/sec if length is in meters and tail-beat frequency is in hertz. Since the dominant schooling species of fish in the Los Angeles Bight are carangiform swimmers, we assume this mode of swimming. Although the entire body of carangiform swimmers takes part in the swimming motion, the maximum excursion occurs between the tip of the caudal fin and the base of the tail. Assuming that any deviations from sinusoidal motion of the tail are of second order, one may write an expression for the maximum speed (\( \dot{r}_o \)) of the fish's tail in a direction perpendicular to the direction of swimming. Thus \( \dot{r}_o \) is given by

\[ \dot{r}_o = 2\pi FA, \]

where \( A \) is the single-sided amplitude of the tail excursion. Working with Bainbridge's data, we find that the average tail-beat peak-to-peak excursion can be expressed as 0.183 times the fish length for all species studied. Consequently, Eq. 2 can be rewritten as

\[ \dot{r}_o = 0.183\pi FL. \]

The Doppler frequency shift expected from a sound scatterer with a speed given by Eq. 3 can be written as

\[ \Delta f = f_o (2\dot{r}_o / c). \]

The maximum frequency shift \( \Delta f \) is expressed in hertz, the term \( f_o \) is the unshifted sonar transmission frequency, and \( c \) is the speed of sound in meters/sec.

Combining Eqs. 1, 3, and 4 results in an expression given in Eq. 5, for the length of the fish in terms of its swimming speed and the maximum observed Doppler shift when the fish presents a side aspect to the sonar beam.

\[ L = 0.65 \frac{c \Delta f}{f_o} - V. \]

The numerical coefficients of \( c \Delta f / f_o \) and \( V \) both have units of time. Thus \( L \) is expressed in meters. In the discussion which follows, the appropriate values of \( c \) and \( f_o \) are 1520 m/sec and 30 000 Hz, respectively. Substitution of these values into Eq. 5 results in

\[ L = 3.3 \times 10^{-2} \Delta f - V. \]

C. Target A

The energy spectrum shown in Fig. 7 is the result of averaging the spectra of three echoes from the data obtained for Target A. The relative positions of the fish school and of the ship at the times of the three echoes indicate a near-broadside aspect on the fish school at the time of each ping.

The dominant return in Fig. 7 is volume reverberation. Since, at side aspect, there is no mean headway component along the sonar beam axis, the echo from the fish
is centered about the same frequency as the volume reverberation and appears in Fig. 7 as a low-level spread of energy about the principal reverberation peak. A comparison of spectrum number 1 in Fig. 3, in which a very low-level target is present, and spectrum number 8 in the same figure, which contains a target of essentially the same strength as in Fig. 7, is of value in determining that the contributions of side lobes of the reverberation peak to frequencies removed by more than 10 Hz from the reverberation center frequency to the fish echo are negligible. Thus, the observed spread of 50 Hz about the reverberation peak can be attributed to motion internal to the fish school. At the mean rate of advance of the school during this period under consideration (0.53 m/sec), even the most extreme deviation (90°) of individual fish headings from the mean school heading could not have accounted for the observed frequency spread. Assuming that the observed Doppler structure was due principally to body and tail swimming motions and that the maximum observed frequency shift was due to peak tail velocity, one can calculate a fish length of 30 cm.

This is in good agreement with the estimate of a 29-cm length derived from the school cruising speed.

D. Target B

The averaged spectra for three echoes from Target B are presented in Fig. 8. The sonar beam intercepted the fish school track at an 18° (±2°) deviation from broadside during the three-ping sequence. This results in a 0.25-m/sec component of headway away from the ship. The average swimming speed of the fish during the sequence was 0.84 m/sec. The largest return in Fig. 8 is volume reverberation. The echo from the fish school is Doppler shifted to a center frequency which is 10 Hz lower than the volume reverberation. The magnitude and direction of the shift are in excellent agreement with the values expected on the basis of the track data. The echo is symmetrically distributed around its peak with a maximum spread of 70 Hz. Substitution of these data into Eq. 6 yields a calculated average fish length of 27 cm. This length is the same as that calculated from school cruising speed.

E. Interpretation of tail aspect Doppler structure

As the target aspect varies from side to head or tail, the Doppler structure rapidly becomes less sensitive to tail and body swimming motions and more sensitive to internal school dynamics. Short-term accelerations associated with individual tail beats, short bursts and periods of slightly slower swimming due to the constantly changing relative positions of fish within a school, and components of tail motion in the forward or aft direction are all short-term motions which can affect the frequency distribution of an echo from a fish school.

While all of the short-term effects mentioned undoubtedly contribute to the Doppler spread at tail aspect, the observed Doppler spread appears to be most readily interpreted in terms of long-term distributions in swimming speeds within a school and changes in those distributions. Three examples of Doppler structure will be cited to illustrate the basis for this hypothesis.
Figures 9 and 10 are averages of seven spectra each from the Target A echoes. Both spectra are taken from a position in the Target A sequence in which the fish presented close to a tail aspect to the sonar beam. A close examination of either spectrum reveals a distribution of echo energy with frequency around the echo peak at about 15 Hz. The distribution appears skewed toward Doppler shifts smaller than the shift of the echo peak. This distribution in frequency corresponds to a similar distribution in motion along the swimming direction of the school. The maximum swimming speed, derived from Fig. 10, is 1.2 m/sec. The minimum speed observed can be estimated as less than 0.3 m/sec, even though the lower limit is masked by volume reverberation. The average swimming speed of the school corresponding to the spectrum in Fig. 10 was 1.1 m/sec.

The skew towards low-speed swimmers persisted over a long interval (see Fig. 4) and is consistent with the hypothesis of an elongation in the fore and aft school dimension during this period, based on the echo level and an 11-kHz sonar measurement of the school dimension along the direction of school travel. A simple calculation based on the differential swimming speed and the duration of the observed skew in speeds indicates that the school elongation during this period of relatively constant cruising was due to some behavioral characteristic shared by all of the school members rather than differences in swimming capabilities or preferences associated with individual fish. If the slow swimmers at echo 21 had maintained that speed through echo 26, and likewise, if the high-speed swimmers had maintained their speed, the school dimension along the direction of travel would have been elongated by more than six times that observed.

A reduction of 40% in the dimension of the school along its direction of travel occurred between pings 26 and 27. An increase in echo level accompanied the apparent compaction. This change in the school was accompanied by a noticeable reduction in the skew of the echo toward the slow-speed scatterers. These observations would indicate a change in swimming behavior in which the compaction of the school was achieved by the slower-swimming fish increasing their speed rather than the faster fish in the leading edge slowing down. The narrowing of the distribution of swimming speeds was followed approximately 6 min later by a high-speed swimming burst.

The third illustration of the impact of swimming behavior on tail or head aspect Doppler structure is taken from the sequence of echoes obtained from Target C. Spectrum No. 3 from the sequence illustrated in Fig. 6 is enlarged and presented in Fig. 11. This spectrum is typical of the 45-min-long head aspect data set. The peak near -90° Hz is bottom reverberation. The largest peak is volume reverberation. The arrow indicates the projected target Doppler shift, based on the school mean swimming speed of 0.45 m/sec. The minimum swimming speed is approximately 0.1 m/sec while the maximum is almost six times greater at 0.58 m/sec. This distribution exhibits a pronounced peak about a frequency shift corresponding to 0.17-m/sec swimming speed. Since the pattern persisted throughout a 45-min period, and since the school dimension did not vary substantially during this period, the observed distributions and their extremes are taken as evidence of a behavioral pattern of swimming. Although there is little direct, documented evidence of such behavior in the open ocean, the sustained pattern of the Doppler structure in echoes from Target C is qualitatively consistent with the accelerations and subsequent gliding associated with filter feeding observed in several species of pelagic fish under laboratory conditions. These species include jack mackerel and anchovy, both of which are common in the Los Angeles Bight.

IV. SUMMARY AND CONCLUSIONS

The internal dynamics of three schools of pelagic fish was investigated, with 1/2-sec acoustic pulses from a 30-kHz sonar. Echoes resulting from these transmissions were analyzed to determine the frequency distribution of echo energy. The observed frequency distributions were interpreted as the result of Doppler shifts due to motion in the fish school. Observed Doppler structure was divided into that caused by body and tail related swimming motions, near-side aspect, and to behavioral swimming characteristics, near-tail or -head aspect. Side aspect data were used to estimate fish length by use of a simple mathematical model. The estimates so derived were consistent with the estimates of fish length made on the basis of school swimming speeds. Tail and head aspect Doppler structure data were correlated with echo level, school dimensions, and school swimming speed to demonstrate that the observed Doppler structure could have resulted from special swimming behavior. In two cases the behavior changed the school dimensions, and in a third situation no change in school dimensions was observed.

The data and interpretations reported in this paper indicate two new applications of narrow-band, active sonar to the field of fisheries biology. Both involve the Doppler effect.

The first application is the accurate measurement of the direction and magnitude of maximum Doppler shift
of volume reverberation relative to the sonar transmission frequency. This measurement allows an accurate estimate of the sonar platform motion relative to the water column. With this information in hand, a sonar with a high range resolution can be used to obtain an accurate track for a school of fish. This track and its first time derivative, the school swimming speed profile, both contain information of interest to the fisheries biologist, such as fish size and swimming endurance.

The second application of the Doppler effect is the direct measure of the spread of echo energy in frequency due to internal school dynamics. Information on swimming, schooling behavior, and fish size is contained in these data.

Either or both of the indicated applications of narrowband sonar may offer an independent check or viable alternatives to currently available methods of remote sensing of average individual fish sizes within a school. It seems reasonable to caution the reader against weighting too heavily the excellent agreement between lengths derived from cruising speeds and similar estimates based on Doppler spread. While the agreement is encouraging, data are presented for only two schools. Further, the only true test of the acoustic technique will be in direct sampling.

The simple mathematical model presented in Sec. III relates maximum observed Doppler shift, school speed, and average fish length. A simple test of the sensitivity of the length estimate provided by this model indicates a strong dependence on the slope and intercept of Bainbridge's equation relating school swimming speed, fish length, and tail-beat frequency (Eq. 1). The length estimate is also quite sensitive to the coefficient which relates tail-beat amplitude and fish length during steady cruising. Equation 5 indicates a severe, but with care attainable, measurement accuracy requirement for the maximum Doppler spread and school speed.

The sensitivity of the model to its measured parameters can be viewed in two ways. First, a significant effort will be necessary in sea-going applications to ensure that the required accuracy is achieved in the presence of sonar platform motions. An optimistic view, however, of the sensitivity of length estimates to Bainbridge's measured parameters is that the model may be sufficiently sensitive to allow resolution of species-specific differences, if they exist. In either case, while resource and time limitations precluded capture of the schools investigated and thus the direct validation of the acoustically derived length estimates, it is apparent from the data presented that such validation is now appropriate.

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6See Ref. 3, p. 123.